Conclusion

Lambda for air at atmospheric conditions has been evaluated based upon what is deemed the best available data. However, these data are not as reliable as they might be and future resolution of the discrepancies in sound-absorption measurements should improve the accuracy of the computations.

Acknowledgment

Concepts reported in this Note are included in the author's dissertation at Colorado State University. The guidance of Professor Virgil A. Sandbrorn and other members of the Committee is appreciated.

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Thermal Response of a Dynamically Loaded Viscoelastic Rod with Variable Properties

Mehdi Pourazady* and Harish Krishnamurty†
University of Toledo, Toledo, Ohio

Abstract

THERMAL response of a viscoelastic rod subject to high-frequency cyclic loading is studied. The material considered models a solid rocket propellant with temperature-dependent thermal conductivity. The mathematical model takes the form of a set of nonlinear, coupled partical differential equations. An algorithm based on an iterative finite-difference method and an averaging technique have been developed to solve these coupled equations directly. The results indicate that the temperature of a material with temperature-dependent thermal conductivity is higher than that of one with constant thermal conductivity. This shows that noticeable errors can occur, especially at the thermal resonant frequencies where thermal conductivity is considered constant.

Viscoelastic materials are dissipative in nature. They dissipate large amounts of mechanical energy in the form of heat when subjected to high-frequency loading. Solid propellant, which is used as the rocket fuel, is also a viscoelastic material. Due to the high speed of the rocket, the solid fuel is subjected to high-frequency vibration. Heating due to vibration near a resonance frequency may lead to melting or material failure. Tormey and Britton[1] conducted vibration tests on solid fuel. They found that the heating due to vibration increased the material temperature significantly to the extent that it even flowed out of the motor. Henter[2] derived a set of coupled partial differential equations for the propagation of stress, strain, and temperature distribution in a viscoelastic media. Huang and Lee[3] studied longitudinal oscillations of viscoelastic rods. They solved the nonlinear coupled equations by iteration, thus determining stress and temperature distributions along the rod. Mukherjee[4] solved the same problem by a simpler method. He substituted a partially linear equation for the nonlinear differential equation and then solved this equation by the finite-difference method. In all the papers cited certain parameters such as thermal conductivity and specific heat, which are weak functions of temperature, are assumed to be constants. This assumption, however, can lead to significant errors, especially at higher temperatures and critical frequencies.[5,6] This is due to the strong temperature dependence of the mechanical properties, which makes the system of equations very sensitive to temperature variations. In this paper the effects of temperature-dependent thermal conductivity on the temperature distribution of a viscoelastic rod subjected to cyclic loading are discussed. Numerical solutions are obtained by the finite-difference method over a wide range of frequencies. The effects of frequency on the temperature and dissipated energy are studied. This leads to a critical frequency at which the temperature is maximum. Heating due to vibration near a critical frequency may cause the solid fuel to soften and melt the material. Therefore, accurate evaluation of these critical frequencies is vital. To simulate the solid rocket propellant, a viscoelastic rod of length 1 insulated on its lateral surface, as shown in Fig. 1, is considered. The left end is free, while the right end is attached to a vibrator that has a prescribed stress given by $\sigma=\sigma_0 \cos\omega t$ where σ_0 is the stress amplitude, ω the frequency, and t the time. The temperature of the vibrator is assumed constant at T_0, a convective boundary condition is assumed at $x=0$, H is the surface conductance, and K is the thermal conductivity of the material. It is also assumed that the solid rocket propellant is a *thermohelogically* simple material. The objective is to find the temperature distribution along the rod. Solving the governing equations (the energy balance equation, equation of motion, and stress-strain relationship),[7] they reduce to the form

$$\frac{d^2\sigma_1}{dx^2}+\omega^2\rho(J_1\sigma_1+J_2\sigma_2)=0 \tag{1}$$

$$\frac{d^2\sigma_2}{dx^2}+\omega^2\rho(J_1\sigma_2-J_2\sigma_1)=0 \tag{2}$$

$$\rho C\left(\frac{\partial T}{\partial t}\right)=\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\omega}{2}J_2(\sigma_1^2+\sigma_2^2) \tag{3}$$

where ρ is the mass density, C the specific heat, J_1 the storage modules, J_2 the loss modules, and σ_1 and σ_2 the real and imaginary parts of stress amplitude σ, respectively. By integrating over a cycle for a periodic motion, the thermomechanical terms due to the oscillation of the rod do not appear in the modified energy equation. The heat dissipated is

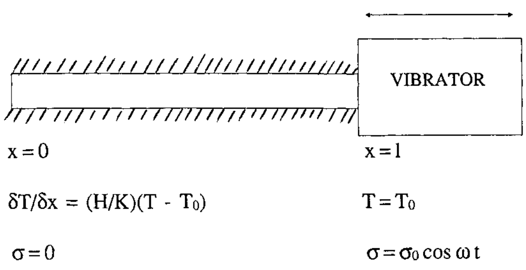

Fig. 1 Model of the solid rocket propellant.

Received Feb. 1, 1988; revision received Feb. 20, 1989.

*Assistant Professor, Mechanical Engineering Department. Member AIAA.

†Graduate Student, Mechanical Engineering Department.

then expressed through the loss compliance J_2 and is given by the term $0.5\omega J_2(\sigma_1^2+\sigma_2^2)$.

The heat-dissipated term represents the work done by the stress on a deformation that is out of phase with the stress and is not recoverable. This is lost as heat. Since it is directly proportional to σ_1^2 and σ_2^2 the heat dissipation of the rod is strongly sensitive to the stress and, therefore, the stress amplitude σ_0. The slightest change in the stress amplitude could cause large variations in the dissipation of heat, and hence large changes in the temperatures along the rod. In this section the energy equation for a material with a temperature-dependent thermal conductivity is derived. From the generalized curve for the temperature dependence of thermal conductivity of the polymers,[8] the following equation for the thermal conductivity of solid rocket propellant is obtained where $C_1=0.01973$, $C_2=1.442\times 10-5$, and $T(^\circ\mathrm{F})$ is the temperature:

$$K=C_1-C_2T \tag{4}$$

Substituting K in Eq. (3), we get

$$C_1\frac{\partial^2 T}{\partial x^2}-C_2\frac{\partial}{\partial x}\left(T\frac{\partial T}{\partial x}\right)+\frac{\omega}{2}J_2(\sigma_1^2+\sigma_2^2)=0 \tag{5}$$

Now consider the term $T^2/2$. Differentiating with respect to x, we get

$$\left(\frac{\partial}{\partial x}\right)\left(\frac{T^2}{2}\right)=T\frac{\partial T}{\partial x} \tag{6}$$

Finally Eqs. (5) and (6) are combined to simplify the energy equation in the following form:

$$C_1\frac{\partial^2 T}{\partial x^2}-C_2\frac{\partial U^2}{\partial x^2}+\frac{\omega}{2}J_2(\sigma_1^2+\sigma_2^2)=0 \tag{7}$$

where $U=T^2/2$. The two sets of Eqs. (1-3) and (1), (2), and (7), are solved separately. The two solutions are then compared to find the effect of temperature-dependent thermal conductivity on temperature distribution. The following data for a solid rocket propellant as given in Ref. 3 are used:

$J^*=(4.61+i1.62)10^{-11}\ \omega^{-0.214}\ (T+125)^{3.21}$ in.^{2}/lb, $\rho l^2=1.023\times 10^3$ psi/s^2, $\rho C=160$ lb/in./in °Fs, $(H/K)l=1.0$, $l=3..$ in, $T_0=65$, °F

The problem is solved numerically by an iterative finite-difference method. The solution procedure is summarized by the following three steps:

Step 1

In this step, Eqs. (1) and (2) are solved simultaneously. This is done iteratively to determine the stress distribution σ_1 and σ_2 along the rod. The temperature values used for evaluation of the stress are from the previous iteration (or the initial guess for the first iteration).

Step 2

This step involves the solutions of Eqs. (3) (constant thermal conductivity) and (7) (variable thermal conductiity). The temperature distribution along the rod is determined using the values of the stresses computed in stage 1. This is also done iteratively until an acceptable convergence is reached.

Step 3

In the final step, the new temperature values calculated in stage 2 are compared with those obtained from the previous iteration (or the initial guesses for the first iteration). If the error is not acceptable, the old temperature values are replaced with the new values and the entire process is iterated until an acceptable level of accuracy is achieved. At the critical frequency convergence is found to be slow or impossible. To overcome this problem, instead of using the new trial temperature found in stage 2, a percentage of this temperature is added to the previous iteration value (*weighting technique*).

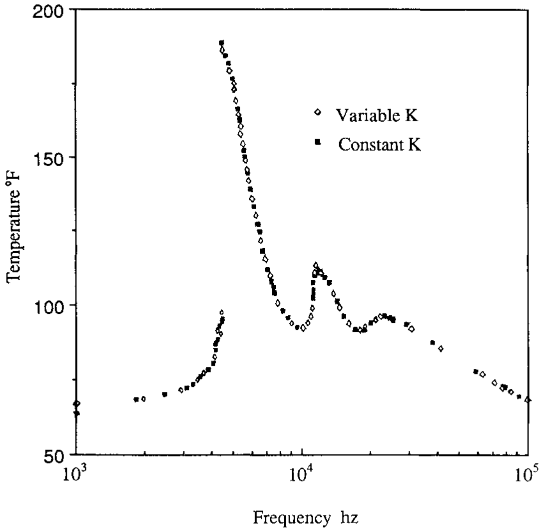

Fig. 2 Peak temperature vs frequency; constant K and variable K, $\sigma_0=1.42$ lb/in.^{2}

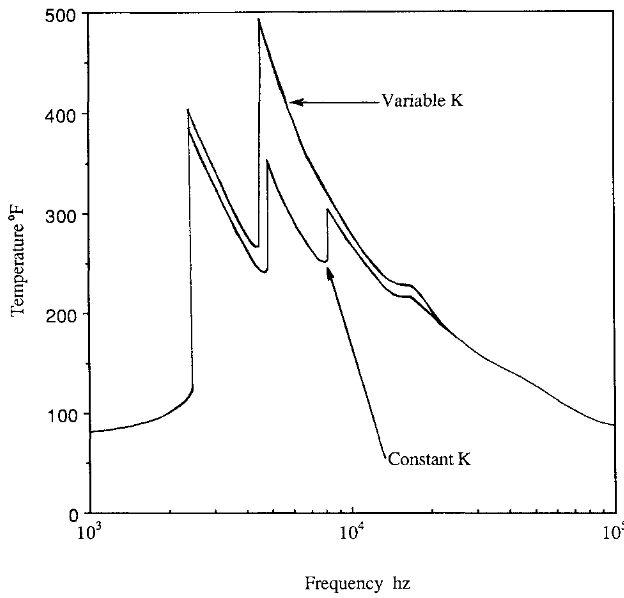

Fig. 3 Peak temperatues vs frequency; constant K and variable K, $\sigma_0=2.84$ lb/in.^{2}

Twenty grid points are chosen along the viscoelastic rod. These points were chosen such that doubling the number of grid points resulted in a change of only 0.2% in the solution. The end conditions are known. Hence, 18 unknowns are present. These equations can be solved by a number of different methods such as Gaussian elimination, LU decomposition, etc. Since the system of equations is represented by a tridiagonal matrix, it is solved by the Thomas algorithm, which is a highly efficient tridiagonal solver.[10]

Figures 2 and 3 show the effect of thermal conductivity on the temperature at two different stress amplitudes. At a low stress amplitude (Fig. 2) the peak temperaturs obtained for temperature-dependent thermal conductivity were found to be very close to the one obtained for constant thermal conductivity. However, at a higher stress amplitude (Fig. 3) the peak temperatures were much higher for temperature-dependent thermal conductivity. For example, for a stess amplitude of 2.84 lb/in.[2], an increase of about 40% in the peak temperature was found at the second thermal resonance frequency. At the thermal resonant frequencies, very sharp increases in temperatures were found. Also, when the constant and variable thermal conductivity solutions were compared, (both solved by the weighing technique) no significant shift in the thermal resonant frequency was observed. This is evident in Fig. 3.

Thermal conductivity of a solid rocket propellant is generally a weak function of temperature. This is perhaps the reason for the constant thermal conductivity assumption in most research efforts studying the phenomenon. However, it is found that at high temperatures, this assumption introduces errors of large magnitudes. Convergence was found to be much slower when thermal conductivity was considered as a function of temperature than when it was fixed. The temperatures were generally higher than those for constant thermal conductivity, especially at the thermal resonant frequencies.

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Analytical Dynamic Model Improvement Using Vibration Test Data

Fu-Shang Wei*
Kaman Aerospace Corporation, Bloomfield, Connecticut

Introduction

THE structural dynamic model of a modern air vehicle becomes critical as the mission requirement becomes more stringent. Ground vibration tests are often used to improve the analytical dynamic model.[1-6]

The mode shapes and natural frequencies obtained from incomplete modal tests do not usually satisfy the dynamic equation and orthogonality requirements. The most common approach is first to modify the analytical mass matrix to satisfy the orthogonality condition based on the measured modal data. Then, the stiffness matrix is modified to fulfill the eigenvalue equation as a function of the measured mode shapes, natural frequencies, and the corrected mass matrix.[7-13]

An alternate approach is to correct the stiffness matrix using static test data. Then, the analytical mass matrix is modified to fulfill the eigenvalue equation based on the modal test data and the updated stiffness matrix.[14]

Both approaches have the merits of improving the analytical structural dynamic models; however, the interaction between mass and stiffness matrices are not taken into consideration in deriving the analytical equations. In this paper, both the analytical mass and stiffness matrices are modified simultaneously using the vibration test data.[15] The dynamic model improvement is obtained using the element correction method combined with the Lagrange multiplier technique.[16] The dynamic equation and the orthogonality constraints are enforced during the analytical derivation. The effects due to mass and stiffness interaction are clearly determined from the final equations. This method is a viable technique for improving an analytical model based on an incomplete set of test data.

Theoretical Formulation

The mode shapes and natural frequencies obtained from ground vibration tests are often incomplete. The desired mass and stiffness matrices are required to be modified to fulfill the eigenvalue equation and the orthogonality constraints using the modal test data.

The measured modal matrix Φ $(n \times m)$ is rectangular, where $n \geq m$, and the natural frequencies matrix Ω^2 $(m \times m)$ is diagonal. Both analytical mass M_A $(n \times n)$ and stiffness K_A $(n \times n)$ matrices are symmetric. The normalized mode shapes and the measured frequencies are assumed correct and have to satisfy the basic orthogonality requirement and eigenvalue equation, as shown in Eqs. (1) and (2).

$$\Phi^T M \Phi = I \tag{1}$$

$$K\Phi = M\Phi\Omega^2 \tag{2}$$

Where $M(n \times n)$ and $K(n \times n)$ are symmetric matrices and represent the corrected mass and stiffness matrices, respectively,

$$M = M^T \tag{3}$$

$$K = K^T \tag{4}$$

It is physically reasonable and mathematically convenient to correct the mass and stiffness matrices, which are subjected to the constraint Eqs. (1-4) by minimizing the weighted Euclidean norms, ϵ_1 and ϵ_2.

$$\epsilon_1 = \tfrac{1}{2}\,||A^{-1/2}(K-K_A)A^{-1/2}|| = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\sum_{q=1}^{n} a_{iq}^{-1/2}\sum_{p=1}^{n}\left(k_{qp}-k_{A_{qp}}\right)a_{pj}^{-1/2}\right]^2 \tag{5}$$

$$\epsilon_2 = \tfrac{1}{2}\,||B^{-1/2}(M-M_A)B^{-1/2}|| = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\sum_{q=1}^{n} b_{iq}^{-1/2}\sum_{p=1}^{n}\left(m_{qp}-m_{A_{qp}}\right)b_{pj}^{-1/2}\right]^2 \tag{6}$$

where matrices $A(n \times n)$ and $B(n \times n)$ are symmetric weighting functions for stiffness and mass matrices, respectively.

Received May 16, 1988; revision received Dec. 20, 1988.

*Senior Aeromechanics Engineer, Test and Development Dept. Member AIAA.